where z is a general independent variable, by expanding the left-hand side of the equation in ascending powers of τ and then equating the coefficients of like terms from both sides of the equation. Having carried out this process, we find that the rth coefficient of the series solution for $\bar{V}/V_0$ is

$$q_r = \frac{(i\omega)^r}{r!}\frac{\theta_0 U_2}{v_2 a_2^r}\left[\frac{(1 - i\omega hc/U_2) + r}{G - (-1)^r H}\right]\exp(i\phi)$$

After some rearrangement, we are then able to obtain the following expression from (15) for the unsteady pressure perturbation at a general fixed point $x = U_2\tau$ on the surface of the wedge:

$$\bar{p}_p/\rho_2 a_2 v_2 = L(\theta_0 U_2/v_2)\{[1 - i\omega hc/U_2 + i\omega x/U_2][1 - N\exp(-2i\omega x/U_2)]\}\exp(i\omega t) \quad (18)$$

where

$$L = \frac{1 + (H/G)^2}{1 - (H/G)^2} \qquad N = \frac{2(H/G)}{1 + (H/G)^2}$$

Now

$$\exp\left(-\frac{2i\omega x}{U_2}\right) = 1 - \frac{2i\omega x}{U_2} + O\left[\left(\frac{\omega x}{U_2}\right)^n\right] \qquad n = 2, 3, \ldots$$

therefore,

$$\frac{\bar{p}_p}{\rho_2 a_2 v_2} \doteq L\frac{U_2}{v_2}\left\{\left[\theta + \dot{\theta}\left(\frac{x - hc}{U_2}\right)\right] - N\left[\theta - \dot{\theta}\left(\frac{x + hc}{U_2}\right)\right]\right\} \quad (19)$$

If we neglect the secondary wave reflections, then $L = 1$, $N = 0$, and Eq. (19) reduces to the simple plane wave relation used in previous applications of the piston theory (see, for example, Lighthill[5] and Miles[2]).

Now the aerodynamic stiffness is given by

$$-m_\theta = \frac{1}{\rho_\infty U_\infty{}^2 c^2}\left(-\frac{\partial M}{\partial \theta}\right)_{\theta=\dot{\theta}=0} \quad (20)$$

and the aerodynamic damping by

$$-m_{\dot{\theta}} = \frac{1}{\rho_\infty U_\infty c^3}\left(-\frac{\partial M}{\partial \dot{\theta}}\right)_{\theta=\dot{\theta}=0} \quad (21)$$

where

$$-M = 2\int_0^c (x - hc)\,\bar{p}_p dx \quad (22)$$

Therefore, from Eqs. (19–22) we obtain

$$-m_\theta = 2L(\rho_2 a_2 U_2/\rho_\infty U_\infty{}^2)[(\tfrac{1}{2} - h)(1 - N)] \quad (23)$$

$$-m_{\dot{\theta}} = 2L(\rho_2 a_2/\rho_\infty U_\infty)[(\tfrac{1}{3} - h + h^2) + N(\tfrac{1}{3} - h^2)] \quad (24)$$

For strong shocks, the coefficients in front of the square brackets in Eqs. (23) and (24) reduce to simple functions of γ and δ, since

$$\frac{U_2}{U_\infty} = 1 \qquad \frac{2\rho_2 a_2}{\rho_\infty U_\infty} = (\gamma + 1)\left(\frac{2\gamma}{\gamma - 1}\right)^{1/2}\delta$$

$$\lambda = \mu = 1 \qquad M = \left(\frac{\gamma - 1}{2\gamma}\right)^{1/2}$$

Concluding Remarks

In Fig. 1, East's experimental values of $-m_\theta$ and $-m_{\dot{\theta}}$ are compared with the theoretical predictions given by Eqs. (23) and (24), which include the effect of the secondary reflections, and also with the equivalent expressions that neglect the reflections, i.e., $L = 1$, $N = 0$. It can be seen that the agreement is generally improved when the reflections are taken into account and that their effect, even for moderate strength shock waves, is not insignificant. However, the value of the "reflection coefficient" N can never be large enough for $m_{\dot{\theta}}$ to be positive even when $M_\infty = \infty$. Therefore, it appears that the major part of the remaining discrepancy between theory and experiment is due to departures from the ideal two-dimensional flow over the model as suggested by East.

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Injection Thrust Termination and Modulation in Solid Rockets

RIAD JAROUDI* AND ALLAN J. MCDONALD†
Thiokol Chemical Corporation, Brigham City, Utah

Nomenclature

A = area, in.^{2}
C^* = characteristic velocity, fps
C_D = discharge coefficient, dimensionless
h = enthalpy, Btu/lb
ΔH_v = latent heat of vaporization, Btu/lb
P = pressure, psia
r = burning rate, in./sec
R = gas constant, ft/°R
T = temperature, °R
V = volume, in.^{3}
W = weight, lb
y = weight fraction of vaporized water, dimensionless
Ω = defined in denominator of Eq. (6)
γ = specific heat ratio, dimensionless
ρ = density, lb/in.^{3}
θ = time, sec

Subscripts

c = chamber
g = propellant gas
p = solid propellant
s = steam
w = water
$(\ )'$ = conditions after injection
1, 2 = time in transient period

RECENT studies by the authors indicate that positive thrust termination and a degree of thrust modulation can be achieved in solid rocket motors by water injection.

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* Senior Engineer, Applied Studies Department, Propulsion Systems Analysis Division; now Senior Engineer, Reaction Motor Division, Denville, N. J.

† Senior Engineer, Applied Studies Department, Propulsion Systems Analysis Division. Member AIAA.

Most existing schemes of thrust termination for solid rocket motors require venting the chamber, resulting in case destruction. Many thrust modulation methods also require extremely heavy, complicated hardware to achieve a variable discharge area. Equations have been derived for the steady-state chamber temperature and pressure following the injection of a coolant into the chamber. An approximate equation has also been derived for the magnitude of the transient period during water injection.

Chamber Temperature and Pressure After Quenching

Combustion extinguishment results from the absorption of large quantities of sensible and latent heat by the injected water. The flow rate of water required for extinguishment varies with the propellant flow rate, propellant gas composition, chamber temperature, and the mass of gases accumulated in the chamber. The injection of high flow rates of water into the chamber results in initial extinguishment by upsetting the mass flow equilibrium conditions in the chamber; complete extinguishment is obtained by cooling the motor, propellant, and residual gases below the propellant autoignition temperature.

Combustion can be temporarily extinguished at lower water flow rates, but reignition may occur if insufficient water is injected to cool the chamber below the propellant autoignition temperature. Injection directly on the propellant surface is not necessary but is advantageous in extinguishing a portion of the burning surface and making combustion extinction easier. Injection of very low flow rates of water modulates thrust by establishing new equilibrium operating conditions in the chamber. The degree of thrust modulation varies with the propellant formulation and flow rate of injected water. Some propellants exhibit various degrees of instability when subjected to rapid temperature and pressure fluctuations.

The equations presented indicate the required flow rate of coolant to achieve any desired chamber condition. Since these equations are nonlinear, an iterative solution for the steady-state chamber temperature and pressure is usually required.

The equations that follow are general and limited by the assumption that the propellant gas and water are thoroughly mixed and the composition of the gas stream in the steady state is constant at all axial points of the combustion chamber.

Heat Balance Equation

$$h_g - h_g' = (\dot{W}_w/\dot{W}_g')\,[y(\Delta H_v') + h_w' - h_w] \quad (1)$$

$$= \text{(for supersaturated steam)}\ (\dot{W}_w/\dot{W}_g') \times (h_s' - h_w) \quad (2)$$

Equations (1) and (2) establish a heat balance between propellant gases and injectant. In addition, the equations are used to determine the steady-state chamber temperature upon which the enthalpies of the equations are dependent.

It is worth noting that, in Eqs. (1) and (2), heat dissipated by the propellant gas is evaluated from the change in the gas enthalpy instead of the gas temperature and specific heat. The former procedure has been found more accurate, since it takes into account the heat effects accompanying changes in the gas composition as the gas temperature decreases.[1]

Chamber Pressure Equation

$$C^* = P_c A_t g/\dot{W}_g \quad (3)$$

$$(C^*)' = P_c' A_t g/(\dot{W}_g' + \dot{W}_w y) \quad (4)$$

Combining Eqs. (3) and (4) and simplifying,

$$P_c'/P_c = (C^*)'\,(\dot{W}_g' + \dot{W}_w y)/C^*\,\dot{W}_g \quad (5)$$

The terms $\dot{W}_g$ and P_c in Eq. (5) can usually be calculated from the rocket geometry and the properties of the propellant. C^* and $(C^*)'$ may be readily calculated from the equation[2]

$$C^* = \frac{(gRT_c)^{1/2}}{\gamma^{1/2}\,[[2/(\gamma+1)]^{\gamma+1/\gamma-1}]^{1/2}} = \frac{(gRT)^{1/2}}{\Omega} \quad (6)$$

$(C^*)'$ is calculated from Eq. (6) by substituting T_c' for T_c and using the new value of γ for the gas-steam mixture. The parameter $\dot{W}_g'$ may be calculated from P_c, P_c', $\dot{W}_g$, and the applicable burning rate equation. The burning rate can often be approximated by the following empirical equation:

$$r = a'P_c^n \quad (7)$$

For many modern perchlorate propellants, the Summerfield equation[2] predicts closely the effect of chamber pressure on the burning rate:

$$\frac{1}{r} = \frac{a}{P_c} + \frac{b}{P_c^{1/3}} \quad (8)$$

Hence,

$$\dot{W}_g' = \dot{W}_g\left[\frac{(a/P_c) + (b/P_c^{1/3})}{(a/P_c') + (b/P_c'^{1/3})}\right] \quad (9)$$

Transient Period Following Injection

The equations shown permit the evaluation of steady-state chamber temperature and pressure after injection. However, the presence of a long transient period preceding the steady state may seriously hinder the applicability of water injection for thrust termination. Hence, it is important to evaluate the magnitude of the transient period.

At any time interval after injection, the following mass balance equation may be written:

$$W_2 = W_1 - ({}_{\text{dis}} - \dot{W}_{\text{gen}})\Delta\dot{W}\theta \quad (10)$$

The terms of Eq. (10) are defined by these equations:

$$\dot{W} = PV/12\,RT \quad (11)$$

$$\dot{W}_{\text{dis}} = gA_t\bar{P}_c/\bar{C}^* \quad (12)$$

$$\dot{W}_{\text{gen}} = \dot{W}_g' + y\dot{W}_w \cong a'(\bar{P}_c)^n\rho_p A_p + C_D A_w y[24\,g\rho_w\,(P_w - \bar{P}_c)]^{1/2} \quad (13)$$

Combining Eqs. (10–13), and simplifying,

$$P_2 = \left(\frac{C_2^*\Omega_2}{C_1^*\Omega_1}\right)^2\left(\frac{P_1V_1}{V_1+\Delta V}\right) - \frac{12(C_2^*\Omega_2)^2\Delta\theta}{g(V_1+\Delta V)} \times \left\{\frac{gA_t\bar{P}_c}{\bar{C}^*} - a'(\bar{P}_c)^n\rho_p A_p - C_D\,A_w y\,[24g\rho_w(P_w - \bar{P}_c)]^{1/2}\right\} \quad (14)$$

Equation (14) will give an accurate solution of the magnitude of the transient period if combustion continues and the intervals ($\Delta\theta$) chosen are sufficiently small.

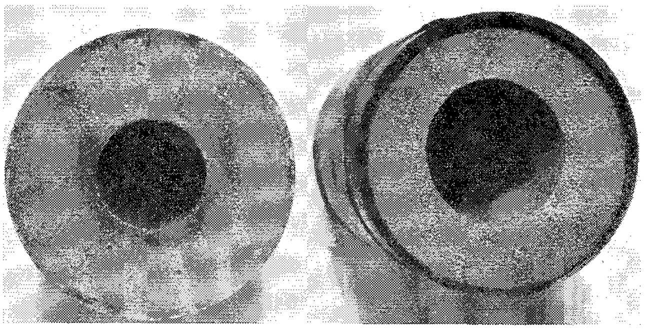
An evaluation of the transient pressure decay if combustion terminates may be obtained from this equation:

$$P_2 = \left(\frac{C_2^*\Omega_2}{C_1^*\Omega_1}\right)^2 P_1 - \frac{12(C_2^*\Omega_2)^2\Delta\theta}{gV_1} \times \left\{\frac{gA_t\bar{P}_c}{\bar{C}^*} - C_D A_w y\,[24g\rho_w(P_w - \bar{P}_c)]^{1/2}\right\} \quad (15)$$

Experimental Investigation

The basic feasibility of extinguishing high-energy solid propellant motors by water injection has been demonstrated using much smaller quantities of water than the copious deluges used in the past. Several tests to verify the quantities of water injectant required for thrust termination were conducted with 2-in. cylindrical perforated motors with slotted grains. The results of these tests are illustrated in Fig. 1.

The condition of the propellant grain before and after extinguishment by water injection is shown in Fig. 1. The surface of the grain did show some leaching of the oxidizer; however, reignition tests were successfully conducted with several of the extinguished motors and no change in ignition characteristics were observed. A typical chamber pressure trace for a motor extinguished by water injection is shown in Fig. 2. The predicted and measured water requirements necessary for extinguishment are presented in Fig. 3.



Before firing **After extinguishment**

Fig. 1 Typical segmented 2-in. cylindrical perforated motor extinguished with water injection.

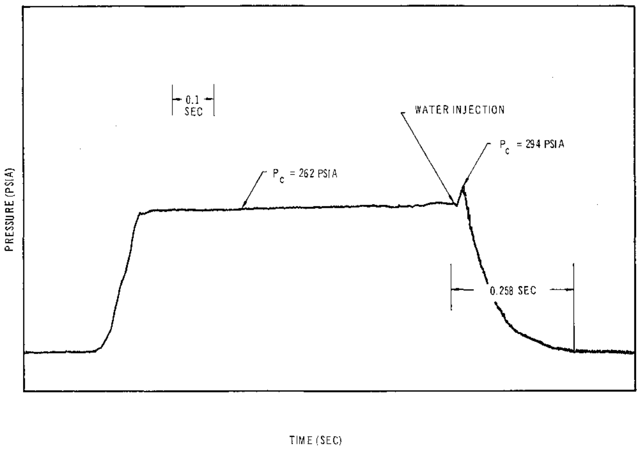


Fig. 2 Typical pressure-time trace of slotted (2-in. cylindrical perforated) motor extinguished with water injection.

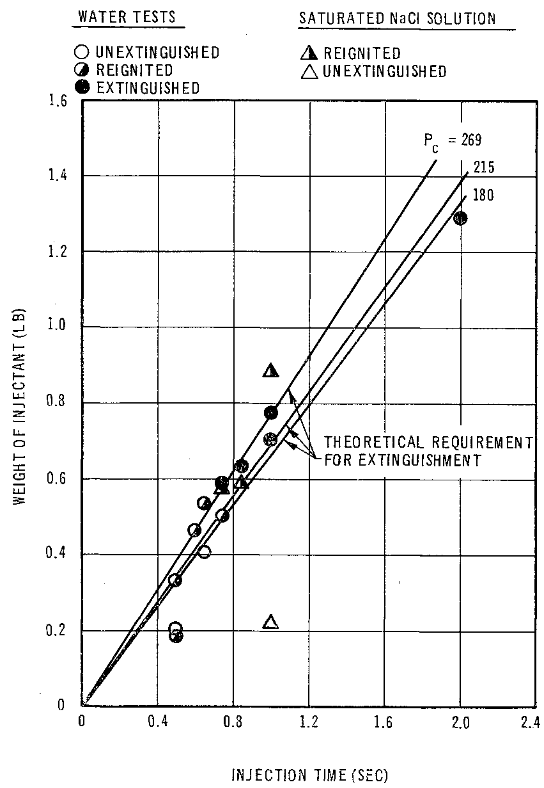


Fig. 3 Injectant weight as a function of injection time for motors operating at 180–269 psia chamber pressure.

The minimum water requirement to obtain complete extinguishment was 0.708 lb H_2O/lb of mixture; this value was in accord with the theoretical weight ratio of 0.715 lb H_2O/lb of mixture. The injection time appears to be critical in that the water must have sufficient residence time in the motor to absorb the residual heat from the propellant gases and propellant surface to prevent reignition.

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Heat Diffusion from Line Source into Mixing Region of Two Parallel Streams

John B. Miles*
University of Missouri, Columbia, Mo.

THE basic problem under consideration is illustrated by Fig. 1. Depicted here are two parallel streams of unequal but uniform velocities interacting in a turbulent mixing region downstream of a thin dividing plate. A line heat source is placed immediately downstream from the dividing plate.

The author's interest in this problem resulted from his investigation of heat transfer for separated flow past relatively deep cavities.[1] The configuration shown in Fig. 1 was one mechanism in the over-all heat-transfer model.

The velocity profile for this configuration has been rather well established[2] to be represented by

$$\Phi = u/u_a = \tfrac{1}{2}[(1 + \Phi_b) + (1 - \Phi_b)\,\mathrm{erf}\eta] \qquad (1)$$

where

u = $u(\eta)$ = x velocity component
u_a = approach velocity of faster moving stream
u_b = approach velocity of slower moving stream
Φ_b = u_b/u_a
η = $\sigma y/x$
σ = $\sigma(\Phi_b)$ = empirical similarity parameter
x, y = coordinates in intrinsic coordinate system, displaced from physical coordinate system (X, Y) by amount η_M

$$\mathrm{erf}\eta = \frac{2}{\pi^{1/2}} \int_0^\eta e^{-\bar{s}^2} ds$$

Equation (1) assumes similar velocity profiles, and the same assumption is employed in the present development for the temperature profiles. η_M is determined from momentum considerations, and the reader is referred to Ref. 2 for the details of this matter. For the mixing of an incompressible fluid with a quiescent wake, the similarity parameter (σ) has been found to be 12. Korst[2] recently has formulated an expression applicable for any value of Φ_b, but there is still much uncertainty about the value of $\sigma(\Phi_b)$.

The differential equation governing the excess temperature (T_{ex}) in the mixing region resulting from the line heat source is

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* Associate Professor of Mechanical Engineering.